channels are simultaneously open. An analogous problem for a different class of *endothermic* reactions was analyzed successfully by Meshkov, Snow, and Yodh,[7] who compared different endothermic reactions at the same outgoing kinetic energy.

In the low-energy region, the rather large $\Sigma^0-\Lambda$ mass difference may cause large deviations from the pure SU_3 predictions, for reactions (1e) and (1f). For example, if tensor forces are important[8] for an incident $^3S_1(\Sigma^- p)$ state, the outgoing $^3D_1$ state of $\Sigma^0 n$ will be strongly suppressed by centrifugal barrier effects relative to the outgoing $^3D_1$ state of Λn.[8]

A particularly interesting comparison may be made between the cross sections for the processes $n+p\rightarrow n+p$ and $\Sigma^+ + p\rightarrow \Sigma^+ + p$. Their $^1S_0$ cross sections both depend only on T_{27} and should be the same. However, the $^3S_1$ cross section for the $\Sigma^+ p$ system depends on T_{10}, whereas the $^3S_1$ system for the $n+p$ system corresponds to the deuteron ($T_{\overline{10}}$). Since

$$\sigma_{\text{tot}}(\Sigma^+\Sigma^+)=(1/4)\sigma^0(\Sigma^+\Sigma^+)+(3/4)\sigma^1(\Sigma^+\Sigma^+) \quad (10)$$

and

$$\sigma^0(nn)=\sigma^0(\Sigma^+\Sigma^+)\,, \quad (11)$$

SU_3 invariance predicts that

$$\sigma_{\text{tot}}(\Sigma^+\Sigma^+)>(1/4)\sigma^0(nn)\,. \quad (12)$$

[7] S. Meshkov, G. A. Snow, and G. B. Yodh, Phys. Rev. Letters **12**, 87 (1964).
[8] D. E. Neville, Phys. Rev. **130**, 327 (1963); J. J. deSwart and C. K. Iddings, *ibid.* **130**, 319 (1963).

The amount by which $\sigma_{\text{tot}}(\Sigma^+\Sigma^+)$ is larger than $\sigma^0(nn)$ is a direct measure of T_{10}. A difficulty with this analysis arises if we consider the hyperon-nucleon potential as arising from meson exchange. The wide variation of the masses of the eight pseudoscalar mesons would imply substantial differences in the ranges of parts of the hyperon-nucleon potential compared to those of the nucleon-nucleon potential.[9] This might produce deviations from the SU_3 prediction given above.

Despite all of the difficulties cited above, comparison of the reactions (1) with Eqs. (2)–(9) should prove useful because it may provide important clues about the effect of SU_3 symmetry breaking on baryon-baryon dynamics. The S-wave cross sections for the reactions Eqs. (1a)–(1g) are all observable, since K^- mesons stopping in a hydrogen bubble chamber provide an excellent source of low-energy Σ^+, Σ^-, and Λ hyperons. The interactions of these hyperons with protons can be studied in the same pictures which record their production.[10]

Note added in proof. Preliminary experimental results of R. Burnstein *et al.*[10] yield $\sigma_{\text{tot}}(\Sigma^+,\Sigma^+)=200\pm100$ mb at a Σ^+ average laboratory momentum of 160 MeV/c. The assumption of SU_3 invariance combined with p-p scattering data predicts $\frac{1}{4}\sigma^0(\Sigma^+,\Sigma^+)=165$ mb at this momentum, indicating that $\sigma^1(\Sigma^+,\Sigma^+)$ is small.

[9] A similar comment has been made by R. H. Dalitz, Proceedings of the Athens Topical Conference, 1963 (unpublished).
[10] R. Burnstein, T. B. Day, B. Kehoe, B. Sechi-Zorn, and G. A. Snow, Bull. Am. Phys. Soc. **8**, 515 (1963); and (to be published).

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Interpretation of High-Energy Large-Angle Scattering*

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It is shown that if the analytically continued partial-wave amplitude is assumed to have l dependence

$$a_{\pm}(s,l)=\sum_{l=0}^{n} C_m{}^{\pm}(s)l^m(l+1)^m$$

for $l<l_0(s)$ and finite n, the scattering amplitude is bounded by $\exp\{-\text{const}[l_0(s)\sin\theta(s)]^{\frac{1}{2}}\}$ at high energies. Here $a_+(s,l)[a_-(s,l)]$ is equal to $a_l(s)$ for even (odd) integer l. The most physical example of this dependence is that in which a central area of the scatterer becomes maximally absorptive.

THE large angle p–p elastic-scattering cross section[1] shows a strong dependence on both energy and momentum transfer. Orear[2] has pointed out that this strong dependence can be fitted by a single exponential in the transverse momentum. If this dependence holds to arbitrarily high energies, the scattering amplitude for a fixed angle must decrease for increasing energy as $\exp(-\text{const } s^{1/2})$, where s is the square of the center-of-mass energy. At any rate it appears that the scattering amplitude for finite fixed angle is a rapidly decreasing function of s.

The purpose of this note is to show that this rapid decrease of the scattering amplitude at finite angles

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[1] G. Cocconi, V. T. Cocconi, A. D. Krisch, J. Orear, R. Rubinstein, D. B. Scarl, W. F. Baker, E. W. Jenkins, and A. L. Read, Phys. Rev. Letters **11**, 499 (1963); W. F. Baker, E. W. Jenkins, A. L. Read, G. Cocconi, V. T. Cocconi, A. D. Krisch, J. Orear, R. Rubinstein, D. R. Scarl, and B. T. Ulrich, Phys. Rev. Letters **12**, 132 (1964).
[2] J. Orear, Phys. Rev. Letters **12**, 112 (1964).

can be understood as a direct consequence of maximal inelasticity of the partial waves whose angular momenta are smaller than a bound $l_0(s)$ which increases with s. We mean by maximal inelasticity that

$$a_l(s)=[\eta_l(s)\exp 2i\delta_l(s)-1]/(2i)\sim i/2$$

as $s\to\infty$ because $\eta_l(s)\to 0$. This is the physically most meaningful interpretation of our result. However, the same effect upon finite angle scattering will be obtained if all $a_l(s)$ go to any allowable fixed limit for $l<l_0(s)$.

More detailed treatment show that even when $l_0(s)$ is relatively small it is impossible for $a_l(s)$ to go to this limit in an l independent way. The diffraction behavior of the full amplitude requires deviation from $a_l(s)=i/2$ which cannot be a too rapidly decreasing function of s. We shall take this l dependence into account by expanding $a_l(s)$ into a power series of l. Since $a_l(s)$ is given only for positive integer values of l, we must discuss how to continue $a_l(s)$ into the full complex plane in order to expand it into a power series. One method is the one used in connection with the Regge formalism. For our purpose, however, we continue $a_l(s)$ into the complex plane by using the definition

$$a_\pm(s,l)=\int_0^1 d(\cos\theta)[T(s,\cos\theta)\pm T(s,-\cos\theta)]P_l(\cos\theta). \quad (1)$$

According to this definition $a_+(s,l)$ equals $a_l(s)$ for even l and $a_-(s,l)$ equals $a_l(s)$ for odd l. Our reason for defining $a_l(s)$ in complex l in this way is that $a_\pm(s,l)$ defined by Eq. (1) has no singularity except at $l=\infty$. Consequently, for the approximation with finite powers of l, the definition (1) is better than any other definition on which $a_l(s)$ might have some singularities at finite l. Because of the symmetry of $P_l(\cos\theta)$ about $l=-\frac{1}{2}$, $a_\pm(s,l)$ has only even power of $(2l+1)$. Therefore we have

$$a_\pm(s,l)\sim\sum_{m=0}^{n} C_m{}^\pm(s)l^m(l+1)^m \quad \text{when} \quad l<l_0(s), \quad (2)$$

where, since $l_0(s)$ will be assumed to increase with s, $C_m{}^\pm(s)$ for $m\neq 0$ must go to zero as a function of s sufficiently rapidly to guarantee convergence of the sum.

Our results are the following: If $a_\pm(s,l)$ is given by Eq. (2) for finite n and the difference between the two sides of Eq. (2) is a sufficiently rapidly decreasing function of s, then the scattering amplitude $T(s,\cos\theta)$ for a finite angle is bounded by

$$|T(s,\cos\theta)|<\exp\{-\text{const}[l_0(s)]^{1/2}\}. \quad (3)$$

The inequality (3) can be extended to small angles depending upon s, as

$$|T[s,\cos\theta(s)]|<\exp\{-\text{const}[l_0(s)\sin\theta(s)]^{1/2}\}. \quad (4)$$

The impact parameter corresponding to $l=l_0(s)$ is $l_0(s)/s^{1/2}$. When $l_0(s)$ increases more slowly than $s^{1/2}$, the scattering in the partial waves is therefore due to a decreasing area of the scatterer. Our result is then: that if the particles are homogeneous over such a small range of impact parameters, $T(s,\cos\theta)$ must decrease rapidly as a function of s. In this note we shall only discuss the inequality (4), since the inequality (3) is a special case of this inequality. In (4) $[l_0(s)\sin\theta(s)]^{1/2}$ might, by improved arguments, be replaced by $l_0(s)\sin\theta(s)$, as we shall discuss later.

In order to prove (4), we use the results of a previous paper.[3] In Paper I we have divided $T[s,\cos\theta(s)]$ into upper and lower sums:

$$T_U[s,\cos\theta(s)] =\sum_{l=0}^{\infty}(2l+1)a_l(s)[1-f_l(s)]P_l[\cos\theta(s)] \quad (5)$$

and

$$T_L[s,\cos\theta(s)]=\sum_{l=0}^{\infty}(2l+1)a_l(s)f_l(s)P_l[\cos\theta(s)] \quad (6)$$

using the "step function"

$$f_l(s)=\{1-\exp[-\alpha\ln^2 s/(l+1)\sin\theta(s)]\}^{\beta\ln s} \quad (7)$$

and have proved that: if $T(s,\cos\theta)$ is analytic in a particular region as a function of $\cos\theta$ and $T(s,\cos\theta)$ is bounded by a power of s at the boundary of the analyticity region, then, if we take α sufficiently large,

$$|T_U[s,\cos\theta(s)]|<s^{-N}, \quad (8)$$

where N is an arbitrary positive number. It should be noted that, by Eq. (7)

$$|1-f_l(s)|<s^{-N} \quad \text{for} \quad l<(N/\alpha)\ln s/\sin\theta(s) \quad (9)$$

and

$$|f_l(s)|<s^{-N} \quad \text{for} \quad l>\alpha_0\ln^2 s/\sin\theta(s), \quad (10)$$

where $\alpha_0=-\alpha/\ln[1-\exp(-N/\beta)]$. The analyticity assumed in I to prove the inequality (8) is that $T(s,\cos\theta)$ is analytic as a function of $\cos\theta$ in an s independent complex neighborhood of the real segment $(-1,1)$ except for its intersection with the cuts from ∞ to $x(s)$ and $-x(s)$ to $-\infty$, $x(s)$ being an arbitrary function of s with $x(s)>1$.

Although it was not remarked in I, the results there are independent of the interpretation of s as the square of the center-of-mass energy provided that s is larger than the center-of-mass energy. If we replace the factor $\alpha_0\ln^2 s/\sin(s)$ in Eqs. (5), (6), and (7) by $l_0(s)$ and call the new step function $F_l(s)$, then the result corresponding to the inequality (8) becomes

$$|T_U[s,\cos\theta(s)]|<\exp-N_0[l_0(s)\sin\theta(s)]^{1/2} \quad (11)$$

with $N_0=N/\alpha_0{}^{1/2}$. The inequalities corresponding to (9) and (10) are

$$|1-F_l(s)|<\exp\{-N_0[l_0(s)\sin\theta(s)]^{1/2}\} \quad (12)$$

[3] K. Yamamoto, Phys. Rev. **134**, B682 (1964), hereafter referred to as I.

for

$$l<(N_0/\alpha)[l_0(s)/\sin\theta(s)]^{1/2}$$

and

$$|F_l(s)|<\exp-N_0[l_0(s)\sin\theta(s)]^{1/2} \quad (13)$$

for

$$l>l_0(s).$$

For the proof of the main inequality (4), it is necessary to obtain a bound for $T_L[s,\cos\theta(s)]$ corresponding to that in the inequality (11) for $T_U[s,\cos\theta(s)]$. If $a_\pm(s,l)$ converges sufficiently rapidly to the form given by Eq. (2) for $l<l_0(s)$, from the inequality (13) it is obvious that the difference between $T_L[s,\cos\theta(s)]$ and

$$T_L{}^0[s,\cos\theta(s)]=\sum_{m=0}^{n}\{C_m{}^+(s)t_m{}^+[s,\cos\theta(s)]+C_m{}^-(s)t_m{}^-[s,\cos\theta(s)]\} \quad (14)$$

is of the order of $\exp\{-\text{const}[l_0(s)\sin\theta(s)]^{1/2}\}$, where

$$t_m{}^+[s,\cos\theta(s)]=\sum_{l=0}^{\infty}(4l+1)(2l)^m(2l+1)^mF_{2l}(s)P_{2l}[\cos\theta(s)] \quad (15)$$

and

$$t_m{}^-[s,\cos\theta(s)]=\sum_{l=0}^{\infty}(4l+3)(2l+1)^m(2l+2)^mF_{2l+1}(s)\times P_{2l+1}[\cos\theta(s)]. \quad (16)$$

In the right-hand side of Eq. (14) we discuss only $t_0{}^+[s,\cos\theta(s)]$, since the same discussion holds for $t_m{}^+[s,\cos\theta(s)]$ with $m\neq 0$ and for $t_m{}^-[s,\cos\theta(s)]$. To estimate $t_0{}^+[s,\cos\theta(s)]$, we consider the following relation:

$$\sum_{l=0}^{\infty}(4l+1)h^{2l}P_{2l}(\cos\theta)=[(1-2h\cos\theta+h^2)^{-3/2}+(1+2h\cos\theta+h^2)^{-3/2}](1-h^2)/2, \quad (17)$$

which can be proved by using the generating function of the Legendre polynomials. If in Eq. (17) $h=1-\exp\{-\text{const}[l_0(s)\sin\theta(s)]^{1/2}\}$, then the right-hand side of Eq. (17) satisfies all the analyticity and bounded properties assumed for the amplitude discussed in I. Therefore, as was shown there, we have the upper sum in Eq. (17)

$$\left|\sum_{l=0}^{\infty}(4l+1)h^{2l}[1-F_{2l}(s)]P_{2l}[\cos\theta(s)]\right|<\exp\{-\text{const}[l_0(s)\sin\theta(s)]^{1/2}\}. \quad (18)$$

Because of the form assumed for h the total sum on the right-hand side of Eq. (17) is of the order of $\exp\{-\text{const}[l_0(s)\sin\theta(s)]^{1/2}\}$, then the lower sum

$$\left|\sum_{l=0}^{\infty}(4l+1)h^{2l}F_{2l}(s)P_{2l}[\cos\theta(s)]\right|<\exp\{-\text{const}[l_0(s)\sin\theta(s)]^{1/2}\}. \quad (19)$$

This discussion is sufficient to prove the inequality (4) because the difference between

$$t_0{}^+[s,\cos\theta(s)] \quad\text{and}\quad \sum_{l=0}^{\infty}(4l+1)h^{2l}F_{2l}(s)P_{2l}[\cos\theta(s)]$$

is again of the order of $\exp\{-\text{const}[l_0(s)\sin\theta(s)]^{1/2}\}$ and exactly the same discussion follows for

$$t_m{}^\pm[s,\cos\theta(s)].$$

Although an ansatz of the form in Eq. (2) is sufficient to obtain our result, we can see that for $l_0(s)$ increasing with s, $C_m{}^\pm(s)$ must go to zero at least $l_0{}^{-2m}(s)$ to guarantee the convergence of Eq. (2). This means that any partial wave goes to the limit $C_0{}^+(s)$ or $C_0{}^-(s)$. It seems reasonable, in view of the increasing number of inelastic channels opening at high energies, that $C_0{}^+(s)$ and/or $C_0{}^-(s)$ goes to $i/2$ as $s\to\infty$ corresponding to maximal inelasticity of the partial wave scattering in the high-energy limit.

It may be possible to replace $\ln^2 s$ in the inequality (10) by $\ln s$.[4] If such an improvement is true, then

$$|T[s,\cos\theta(s)]|<\exp\{-\text{const}\, l_0(s)\sin\theta(s)\} \quad (20)$$

holds instead of the inequality (4). The maximum possible $l_0(s)$ is const $s^{1/2}\ln s$, because we know that[5]

$$|a_l(s)|<P(s)\exp(-\text{const}\, l/s^{1/2}), \quad (21)$$

where $P(s)$ is a polynomial of s. Therefore, if the inequality (20) is correct, the minimum of the scattering amplitude due to maximal partial-wave inelasticity is $\exp[-\text{const}\, s^{1/2}\ln s\sin\theta(s)]$. It is interesting to note that this minimum is equal to that of Cerulus and Martin, and of Kinoshita.[6] Without the $\ln s$ factor in the exponent, this would be just the fit obtained by Orear.[2]

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[4] See the footnote 5 of Ref. 3.
[5] M. Froissart, Phys. Rev. **123**, 1053 (1961).
[6] F. Cerulus and A. Martin, Phys. Letters **8**, 80 (1964) for $\sin\theta$ dependence T. Kinoshita, Phys. Rev. Letters **12**, 257 (1964).